Inverse of a Matrix by Elementary Operations

4 Marks Questions

1. Use elementary column operations $C_2 \to C_2 - 2C_1$ in the matrix equation

$$\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}.$$

Foreign 2014

> We write the matrix A as $A = AI$ for applying elementary column operations. So, apply column operation on the matrix of LHS and on the second matrix of RHS.

Given matrix equation is

$$\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

On applying $C_2 \to C_2 - C_1$, we get

$$\begin{bmatrix} 4 & 2-8 \\ 3 & 3-6 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0-4 \\ 1 & 1-2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 4 & -6 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & -4 \\ 1 & -1 \end{bmatrix}$$

which is the required answer.

2. Using elementary row transformation (ERT), find inverse of matrix $A = \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix}$.

Foreign 2010; HOTS

> Firstly, put $A = IA$. Then, by applying elementary row transformation on A of LHS and I of RHS, convert this matrix in the form $I = BA$, where B gives the inverse of A.

Given matrix is $A = \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix}$.

Let $\qquad A = IA$

$$\Rightarrow \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$ **(1/2)**

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{bmatrix} 1 & 1 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$ **(1)**

Applying $R_2 \to R_2 - 5R_1$, we get

$$\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -5 & 6 \end{bmatrix} A$$ **(1)**

Applying $R_1 \to R_1 + R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ -5 & 6 \end{bmatrix} A$$ **(1/2)**

Now, applying $R_2 \to (-1)R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ 5 & -6 \end{bmatrix} A$$ **(1/2)**

Hence, $A^{-1} = \begin{bmatrix} -4 & 5 \\ 5 & -6 \end{bmatrix}$ $[\because A^{-1}A = I]$ **(1/2)**

3. Find A^{-1}, by using elementary row transformation for matrix $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$.

Foreign 2010

Do same as Que 2. $\left[\textbf{Ans. } A^{-1} = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}\right]$

4. Using elementary row transformation, find inverse of matrix $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.

Delhi 2010

Do same as Que 2. $\left[\textbf{Ans. } A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}\right]$

6 Marks Questions

5. $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Delhi 2012

Given matrix is $A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

Let $A = IA$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad \textbf{(1)}$$

Applying $R_2 \rightarrow R_2 + R_1$, $R_3 \rightarrow R_3 + 3R_1$, we get

$$\begin{bmatrix} -1 & 1 & 2 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A \quad \textbf{(1)}$$

Applying $R_1 \rightarrow (-1) R_1$, we get

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - R_3$, we get

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & -1 & -2 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & 1 & -1 \\ 3 & 0 & 1 \end{bmatrix} A \qquad \textbf{(1)}$$

Applying $R_1 \to R_1 - R_2$ and $R_3 \to R_3 + 4R_2$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -2 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 1 & -1 \\ -5 & 4 & -3 \end{bmatrix} A \qquad \textbf{(1)}$$

Applying $R_2 \to (-1)\, R_2$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 1 \\ -5 & 4 & -3 \end{bmatrix} A$$

Applying $R_2 \to R_2 + 2\, R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ -5 & 4 & -3 \end{bmatrix} A \qquad \textbf{(1)}$$

Applying $R_3 \to (-1)\, R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} A$$

which is of the form $I = BA$.

Hence,

$$A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} \qquad \textbf{(1)}$$

6. $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Foreign 2011

Given matrix is $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$.

Let $A = IA$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad \textbf{(1)}$$

Applying $R_1 \to 3R_1$, we get

$$\begin{bmatrix} 6 & 0 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad \textbf{(1/2)}$$

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{bmatrix} 1 & -1 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad \textbf{(1)}$$

Applying $R_1 \to R_1 + R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad \textbf{(1/2)}$$

Applying $R_2 \to R_2 - 5R_1$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 0 & 0 & 1 \end{bmatrix} A \qquad \textbf{(1)}$$

Applying $R_3 \to R_3 - R_2$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 15 & -6 & 6 \end{bmatrix} A \qquad \textbf{(1/2)}$$

Applying $R_3 \to \frac{1}{3} R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A \qquad \textbf{(1/2)}$$

Hence, $$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \qquad \textbf{(1)}$$

7. $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

Delhi 2010

Given matrix is $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$. **(1)**

Let $A = IA$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ **(1)**

Applying $R_2 \to R_2 + 3R_1$ and $R_3 \to R_3 - 2R_1$, we get

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -7 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$ **(1)**

Applying $R_2 \to \frac{R_2}{9}$, we get

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -7/9 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1/9 & 0 \\ -2 & 0 & 1 \end{bmatrix} A \quad \textbf{(1/2)}$$

Applying $R_3 \to R_3 + 5R_2$, we get

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -7/9 \\ 0 & 0 & 1/9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1/9 & 0 \\ -1/3 & 5/9 & 1 \end{bmatrix} A \textbf{(1/2)}$$

Applying $R_3 \to 9R_3$, we get

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -7/9 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1/9 & 0 \\ -3 & 5 & 9 \end{bmatrix} A \quad \textbf{(1/2)}$$

Applying $R_1 \to R_1 - 3R_2$, we get

$$\begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 1 & -7/9 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1/3 & 0 \\ 1/3 & 1/9 & 0 \\ -3 & 5 & 9 \end{bmatrix} A \textbf{ (1/2)}$$

Applying $R_1 \to R_1 - \frac{1}{3} R_3$ and $R_2 \to R_2 + \frac{7}{9} R_3$,

we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix} A$$

Hence, $$A^{-1} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix} \quad \textbf{(1)}$$

8. $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

All India 2010

Do same as Que 7. $\left[\textbf{Ans. } A^{-1}=\begin{bmatrix}3 & 2 & 6\\1 & 1 & 2\\2 & 2 & 5\end{bmatrix}\right]$

9. $\begin{bmatrix}3 & 0 & -1\\2 & 3 & 0\\0 & 4 & 1\end{bmatrix}$ All India 2009

Do same as Que 7. $\left[\textbf{Ans. } A^{-1}=\begin{bmatrix}3 & -4 & 3\\-2 & 3 & -2\\8 & -12 & 9\end{bmatrix}\right]$

10. $\begin{bmatrix}1 & 2 & 3\\2 & 5 & 7\\-2 & -4 & -5\end{bmatrix}$ All India 2008

Do same as Que 7. $\left[\textbf{Ans. } A^{-1}=\begin{bmatrix}3 & -2 & -1\\-4 & 1 & -1\\2 & 0 & 1\end{bmatrix}\right]$